Section 2.2, WW #9

Object hanging from a spring, in a liquid with damping constant $d > 0$.
Motion is described by the ODE:

$$y'' + dy' + 100y = 0$$

For what values of d do solutions
(a) oscillate with constant amplitude for all time?
(b) Oscillate with decreasing amplitude?
(c) decrease in amplitude without oscillation?

Solution: Characteristic equation:

$$r^2 + dr + 100 = 0$$

Roots: $$r_{1,2} = \frac{-d \pm \sqrt{d^2 - 4 \cdot 1 \cdot 100}}{2}$$

We have oscillations if r_1 & r_2 are complex,
i.e. If $d^2 - 400 < 0$ (negative under the square root).

Then $$r_{1,2} = -\frac{d}{2} \pm \sqrt{400 - d^2}\, i = \alpha \pm i\beta$$

Then $$y(t) = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t).$$

Constant amplitude if $\alpha = -\frac{d}{2} = 0$, so

(a) Oscillations w/ const. ampl. if $d = 0$
(b) Oscillations w/ decreasing amplitude if $d \in (0, 20)$

(b) Oscillations w/ decreasing amplitude if $d > 0$ & $d^2 - 400 < 0$, i.e. $d \in (0, 20)$

(c) No oscillations: $d^2 - 400 \geq 0$ $d \in [20, \infty)$